SGS Models

PALM group

Institute of Meteorology and Climatology, Leibniz Universität Hannover

last update: 21st September 2015

SGS Models (I)

SGS Models (I)

- The SGS model has to parameterize the effect of the SGS motions (small-scale turbulence) on the large eddies (resolved-scale turbulence).

SGS Models (I)

- The SGS model has to parameterize the effect of the SGS motions (small-scale turbulence) on the large eddies (resolved-scale turbulence).
- Features of small-scale turbulence: local, isotropic, dissipative (inertial subrange)

SGS Models (I)

- The SGS model has to parameterize the effect of the SGS motions (small-scale turbulence) on the large eddies (resolved-scale turbulence).
- Features of small-scale turbulence: local, isotropic, dissipative (inertial subrange)
- SGS stresses should depend on:
  - local resolved-scale field and / or
  - past history of the local fluid (via a PDE)

SGS Models (I)

- The SGS model has to parameterize the effect of the SGS motions (small-scale turbulence) on the large eddies (resolved-scale turbulence).
- Features of small-scale turbulence: local, isotropic, dissipative (inertial subrange)
- SGS stresses should depend on:
  - local resolved-scale field and / or
  - past history of the local fluid (via a PDE)
- Importance of the model depends on how much energy is contained in the subgrid-scales:
  - $E_{SGS}/E < 50\%$: results relatively insensitive to the model, (but sensitive to the numerics, e.g. in case of upwind scheme)
  - $E_{SGS}/E = 1$: model more important

SGS Models (I)

- The SGS model has to parameterize the effect of the SGS motions (small-scale turbulence) on the large eddies (resolved-scale turbulence).
- Features of small-scale turbulence: local, isotropic, dissipative (inertial subrange)
- SGS stresses should depend on:
  - local resolved-scale field and / or
  - past history of the local fluid (via a PDE)
- Importance of the model depends on how much energy is contained in the subgrid-scales:
  - $E_{SGS}/E < 50\%$: results relatively insensitive to the model, (but sensitive to the numerics, e.g. in case of upwind scheme)
  - $E_{SGS}/E = 1$: model more important
  - **If the large-scale eddies are not resolved, the SGS model and the LES will fail at all!**

SGS Models (II)

Requirements that a good SGS model must fulfill:

SGS Models (II)

Requirements that a good SGS model must fulfill:

- Represent interactions with small scales.

SGS Models (II)

Requirements that a good SGS model must fulfill:

- Represent interactions with small scales.
- Provide adequate dissipation
(transport of energy from the resolved grid scales to the unresolved grid scales; the rate of dissipation ε in this context is the flux of energy through the inertial subrange).

SGS Models (II)

Requirements that a good SGS model must fulfill:

- Represent interactions with small scales.
- Provide adequate dissipation
(transport of energy from the resolved grid scales to the unresolved grid scales; the rate of dissipation ε in this context is the flux of energy through the inertial subrange).
- Dissipation rate must depend on the large scales of the flow rather than being imposed arbitrarily by the model. The SGS model must depend on the large-scale statistics and must be sufficiently flexible to adjust to changes in these statistics.

SGS Models (II)

Requirements that a good SGS model must fulfill:

- Represent interactions with small scales.
- Provide adequate dissipation
  (transport of energy from the resolved grid scales to the unresolved grid scales; the rate of dissipation ε in this context is the flux of energy through the inertial subrange).
- Dissipation rate must depend on the large scales of the flow rather than being imposed arbitrarily by the model. The SGS model must depend on the large-scale statistics and must be sufficiently flexible to adjust to changes in these statistics.
- In energy conserving codes (ideal for LES) the only way for TKE to leave the resolved modes is by the dissipation provided by the SGS model.

SGS Models (II)

Requirements that a good SGS model must fulfill:

- Represent interactions with small scales.
- Provide adequate dissipation
  (transport of energy from the resolved grid scales to the unresolved grid scales; the rate of dissipation ε in this context is the flux of energy through the inertial subrange).
- Dissipation rate must depend on the large scales of the flow rather than being imposed arbitrarily by the model. The SGS model must depend on the large-scale statistics and must be sufficiently flexible to adjust to changes in these statistics.
- In energy conserving codes (ideal for LES) the only way for TKE to leave the resolved modes is by the dissipation provided by the SGS model.
- The primary goal of an SGS model is to obtain correct statistics of the energy containing scales of motion.

SGS Models (III)

All the above observations suggest the use of an eddy viscosity type SGS model:

SGS Models (III)

All the above observations suggest the use of an eddy viscosity type SGS model:

- Take idea from RANS modeling, introduce eddy viscosity ν_T:

$$\tau_{ki} = -\nu_T \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right) = -2\nu_T \overline{S}_{ki} \qquad \text{with} \quad \overline{S}_{ki} = \frac{1}{2} \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right)$$

filtered strain rate tensor

SGS Models (III)

All the above observations suggest the use of an eddy viscosity type SGS model:

- Take idea from RANS modeling, introduce eddy viscosity ν_T:

$$\tau_{ki} = -\nu_T \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right) = -2\nu_T \overline{S}_{ki} \qquad \text{with} \quad \overline{S}_{ki} = \frac{1}{2} \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right)$$

filtered strain rate tensor

Now we need a model for the eddy viscosity:

SGS Models (III)

All the above observations suggest the use of an eddy viscosity type SGS model:

- Take idea from RANS modeling, introduce eddy viscosity ν_T:

$$\tau_{ki} = -\nu_T \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right) = -2\nu_T \overline{S}_{ki} \qquad \text{with} \quad \overline{S}_{ki} = \frac{1}{2} \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right)$$

filtered strain rate tensor

Now we need a model for the eddy viscosity:

- Dimensionality of ν_T is l^2/t

SGS Models (III)

All the above observations suggest the use of an eddy viscosity type SGS model:

- Take idea from RANS modeling, introduce eddy viscosity ν_T:

$$\tau_{ki} = -\nu_T \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right) = -2\nu_T \overline{S}_{ki} \qquad \text{with} \quad \overline{S}_{ki} = \frac{1}{2} \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right)$$

filtered strain rate tensor

Now we need a model for the eddy viscosity:

- Dimensionality of ν_T is l^2/t
- Obvious choice: $\nu_T = Cql$ (q, l: characteristic velocity / length scale)

SGS Models (III)

All the above observations suggest the use of an eddy viscosity type SGS model:

- Take idea from RANS modeling, introduce eddy viscosity ν_T:

$$\tau_{ki} = -\nu_T \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right) = -2\nu_T \overline{S}_{ki} \qquad \text{with} \quad \overline{S}_{ki} = \frac{1}{2} \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right)$$

filtered strain rate tensor

Now we need a model for the eddy viscosity:

- Dimensionality of ν_T is l^2/t
- Obvious choice: $\nu_T = Cql$ (q, l: characteristic velocity / length scale)
- Turbulence length scale is easy to define: largest size of the unresolved scales is Δ $\quad l = \Delta$

SGS Models (III)

All the above observations suggest the use of an eddy viscosity type SGS model:

- Take idea from RANS modeling, introduce eddy viscosity ν_T:

$$\tau_{ki} = -\nu_T \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right) = -2\nu_T \overline{S}_{ki} \qquad \text{with} \quad \overline{S}_{ki} = \frac{1}{2} \left(\frac{\partial \overline{u_k}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_k} \right)$$

filtered strain rate tensor

Now we need a model for the eddy viscosity:

- Dimensionality of ν_T is l^2/t
- Obvious choice: $\nu_T = Cql$ (q, l: characteristic velocity / length scale)
- Turbulence length scale is easy to define: largest size of the unresolved scales is Δ $\quad l = \Delta$
- Velocity scale not obvious (smallest resolved scales, their size is of the order of the variation of velocity over one grid element)

$$q = l \frac{\partial \overline{u}}{\partial x} = l\overline{S} \qquad \text{for 3D: } \overline{S} = \sqrt{2\overline{S}_{ki}\,\overline{S}_{ki}}$$

characteristic filtered rate of strain

The Smagorinsky Model

The Smagorinsky Model

Combine previous expressions to obtain:

$$\nu_T = C\Delta^2\overline{S} = (C_S\Delta)^2\overline{S}$$

The Smagorinsky Model

Combine previous expressions to obtain:

$$\nu_T = C\Delta^2\overline{S} = (C_S\Delta)^2\overline{S}$$

Model due to Smagorinsky (1963):

- Originally designed at NCAR for global weather modeling.

The Smagorinsky Model

Combine previous expressions to obtain:

$$\nu_T = C\Delta^2\overline{S} = (C_S\Delta)^2\overline{S}$$

Model due to Smagorinsky (1963):

- Originally designed at NCAR for global weather modeling.
- Can be derived in several ways: heuristically (above), from inertial range arguments (Lilly), from turbulence theory.

The Smagorinsky Model

Combine previous expressions to obtain:

$$\nu_T = C\Delta^2\overline{S} = (C_S\Delta)^2\overline{S}$$

Model due to Smagorinsky (1963):

- Originally designed at NCAR for global weather modeling.
- Can be derived in several ways: heuristically (above), from inertial range arguments (Lilly), from turbulence theory.
- Constant predicted by all methods (based on theory, decay of isotropic turbulence): $C_S = \sqrt{C} \approx 0.2$

The Smagorinsky Model: Performance

The Smagorinsky Model: Performance

- Predicts many flows reasonably well

The Smagorinsky Model: Performance

- Predicts many flows reasonably well
- Problems:
  - Optimum parameter value varies with flow type:
    - Isotropic turbulence: $C_S \approx 0.2$
    - Shear (channel) flows: $C_S \approx 0.065$

The Smagorinsky Model: Performance

- Predicts many flows reasonably well
- Problems:
  - Optimum parameter value varies with flow type:
    - Isotropic turbulence: $C_S \approx 0.2$
    - Shear (channel) flows: $C_S \approx 0.065$
  - Length scale uncertain with anisotropic filter:

$$(\Delta_x \Delta_y \Delta_z)^{1/3} \qquad (\Delta_x + \Delta_y + \Delta_z)/3$$

The Smagorinsky Model: Performance

- Predicts many flows reasonably well
- Problems:
  - Optimum parameter value varies with flow type:
    - Isotropic turbulence: $C_S \approx 0.2$
    - Shear (channel) flows: $C_S \approx 0.065$
  - Length scale uncertain with anisotropic filter:

$$(\Delta_x \Delta_y \Delta_z)^{1/3} \quad (\Delta_x + \Delta_y + \Delta_z)/3$$

  - Needs modification to account for:
    - stratification: $C_S = F(Ri, ...)$, Ri: Richardson number
    - near-wall region: $C_S = F(z+)$, $z+$: distance from wall

Deardorff (1980) Modification (Used in PALM) (I)

$$\nu_T = Cql = C_M \Lambda \sqrt{\bar{e}} \quad \textbf{with} \quad \bar{e} = \frac{\overline{u_i' u_i'}}{2} \quad \textbf{SGS-turbulent kinetic energy}$$

Deardorff (1980) Modification (Used in PALM) (I)

$$\nu_T = Cql = C_M \Lambda \sqrt{\bar{e}} \quad \textbf{with} \quad \bar{e} = \frac{\overline{u_i' u_i'}}{2} \quad \textbf{SGS-turbulent kinetic energy}$$

- The SGS-TKE allows a much better estimation of the velocity scale for the SGS fluctuations and also contains information about the past history of the local fluid.

Deardorff (1980) Modification (Used in PALM) (I)

$$\nu_T = Cql = C_M \Lambda \sqrt{\bar{e}} \quad \textbf{with} \quad \bar{e} = \frac{\overline{u_i' u_i'}}{2} \quad \textbf{SGS-turbulent kinetic energy}$$

- The SGS-TKE allows a much better estimation of the velocity scale for the SGS fluctuations and also contains information about the past history of the local fluid.

$$C_M = const. = 0.1$$

$$\Lambda = \begin{cases} min\left(0.7 \cdot z, \Delta\right), & \textbf{unstable or neutral stratification} \\ min\left(0.7 \cdot z, \Delta, 0.76\sqrt{\bar{e}}\left[\frac{g}{\Theta_0}\frac{\partial \bar{\Theta}}{\partial z}\right]^{-1/2}\right), & \textbf{stable stratification} \end{cases}$$

$$\Delta = \left(\Delta x \Delta y \Delta z\right)^{1/3}$$

Deardorff (1980) Modification (Used in PALM) (II)

- SGS-TKE from prognostic equation

$$\frac{\partial \bar{e}}{\partial t} = -\bar{u}_k \frac{\partial \bar{e}}{\partial x_k} - \tau_{ki} \frac{\partial \bar{u}_i}{\partial x_k} + \frac{g}{\Theta_0} \overline{u_3' \Theta'} - \frac{\partial}{\partial x_k} \left\{ \overline{u_k' \left(e' + \frac{\pi'}{\rho_0} \right)} \right\} - \epsilon$$

$$\tau_{ki} = -K_m \left(\frac{\partial \bar{u}_i}{\partial x_k} + \frac{\partial \bar{u}_k}{\partial x_i} \right) + \frac{2}{3} \delta_{ik} \bar{e} \qquad \text{with} \qquad K_m = 0.1 \cdot \Lambda \sqrt{\bar{e}}$$

$$H_k = \overline{u_k' \Theta'} = -K_h \frac{\partial \bar{\Theta}}{\partial x_k} \qquad \text{with} \qquad K_h = \left(1 + 2 \frac{\Lambda}{\Delta} \right)$$

$$W_k = \overline{u_k' q'} = -K_h \frac{\partial \bar{q}}{\partial x_k}$$

$$\frac{\partial}{\partial x_k} \left[\overline{u_k' \left(e' + \frac{\pi'}{\rho_0} \right)} \right] = -\frac{\partial}{\partial x_k} \nu_e \frac{\partial \bar{e}}{\partial x_k}$$

$$\nu_e = 2 \nu_T$$

$$\epsilon = C_\epsilon \frac{\bar{e}^{3/2}}{\Lambda} \qquad C_\epsilon = 0.19 + 0.74 \frac{\Lambda}{\Delta}$$

Deardorff (1980) Modification (Used in PALM) (III)

- There are still problems with this parameterization:

Deardorff (1980) Modification (Used in PALM) (III)

- There are still problems with this parameterization:
  - The model overestimates the velocity shear near the wall.

Deardorff (1980) Modification (Used in PALM) (III)

- There are still problems with this parameterization:
  - The model overestimates the velocity shear near the wall.
  - C_M is still a constant but actually varies for different types of flows.

Deardorff (1980) Modification (Used in PALM) (III)

- There are still problems with this parameterization:
  - The model overestimates the velocity shear near the wall.
  - $\mathrm{C_M}$ is still a constant but actually varies for different types of flows.
  - Backscatter of energy from the SGS-turbulence to the resolved-scale flow can not be considered.

Deardorff (1980) Modification (Used in PALM) (III)

- There are still problems with this parameterization:
  - The model overestimates the velocity shear near the wall.
  - $\mathrm{C_M}$ is still a constant but actually varies for different types of flows.
  - Backscatter of energy from the SGS-turbulence to the resolved-scale flow can not be considered.
- Several other SGS models have been developed:
  - Two part eddy viscosity model (Sullivan, et al.)

Deardorff (1980) Modification (Used in PALM) (III)

- There are still problems with this parameterization:
  - The model overestimates the velocity shear near the wall.
  - $\mathrm{C_M}$ is still a constant but actually varies for different types of flows.
  - Backscatter of energy from the SGS-turbulence to the resolved-scale flow can not be considered.
- Several other SGS models have been developed:
  - Two part eddy viscosity model (Sullivan, et al.)
  - Scale similarity model (Bardina et al.)

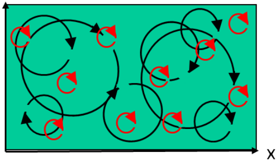
Deardorff (1980) Modification (Used in PALM) (III)

- There are still problems with this parameterization:
  - The model overestimates the velocity shear near the wall.
  - $\mathrm{C_M}$ is still a constant but actually varies for different types of flows.
  - Backscatter of energy from the SGS-turbulence to the resolved-scale flow can not be considered.
- Several other SGS models have been developed:
  - Two part eddy viscosity model (Sullivan, et al.)
  - Scale similarity model (Bardina et al.)
  - Backscatter model (Mason)

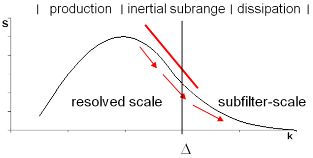
Deardorff (1980) Modification (Used in PALM) (III)

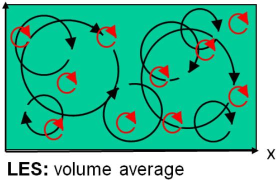
- There are still problems with this parameterization:
  - The model overestimates the velocity shear near the wall.
  - $\mathrm{C_M}$ is still a constant but actually varies for different types of flows.
  - Backscatter of energy from the SGS-turbulence to the resolved-scale flow can not be considered.
- Several other SGS models have been developed:
  - Two part eddy viscosity model (Sullivan, et al.)
  - Scale similarity model (Bardina et al.)
  - Backscatter model (Mason)
- However, for fine grid resolutions ($\mathrm{E_{SGS}} << \mathrm{E}$) LES results become almost independent from the different models (Beare et al., 2006, BLM).

Summary / Important Points for Beginners (I)

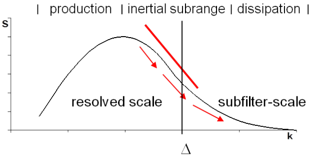


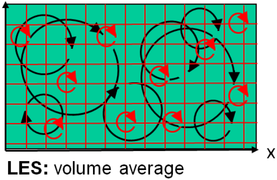
Summary / Important Points for Beginners (I)



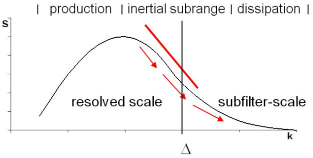


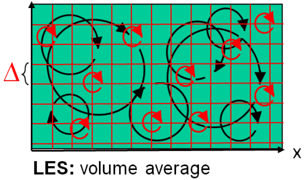
Summary / Important Points for Beginners (I)



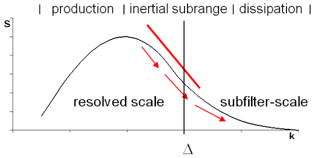


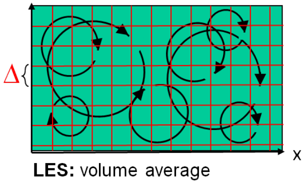
Summary / Important Points for Beginners (I)



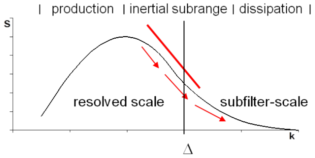


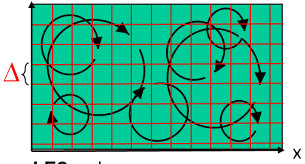
Summary / Important Points for Beginners (I)



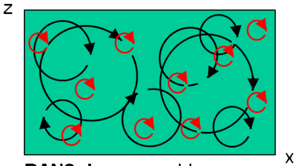


Summary / Important Points for Beginners (I)



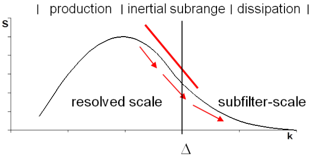


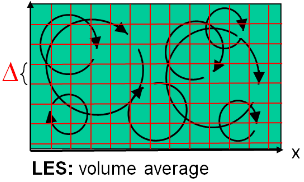
LES: volume average

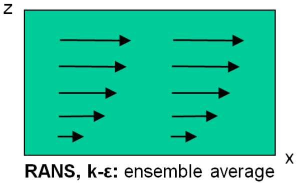


RANS, k-ε: ensemble average

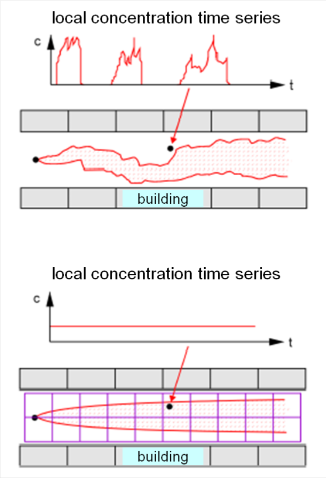
Summary / Important Points for Beginners (I)



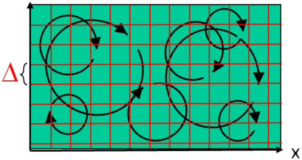




Summary / Important Points for Beginners (I)

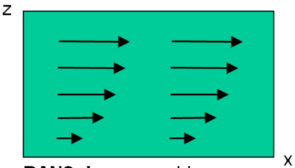


fluctuations
(**u**,c)



LES: volume average

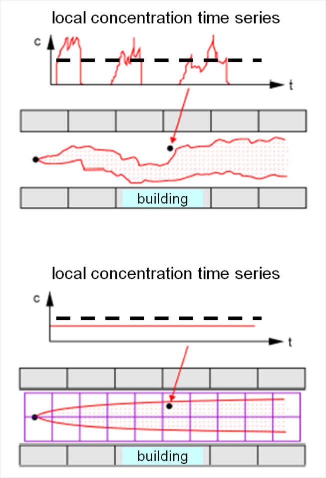
smooth result



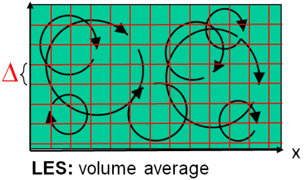
RANS, k-ε: ensemble average

after Schatzmann and Leitl (2001)

Summary / Important Points for Beginners (I)

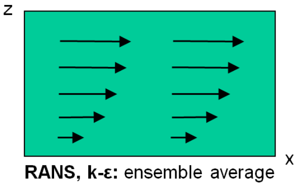


fluctuations
(**u**,c)



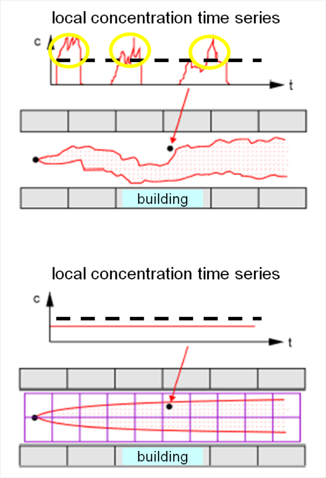
_ _ _ critical concentration level

smooth result

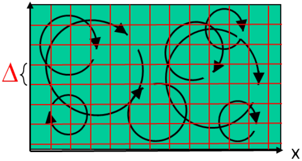


after Schatzmann and Leitl (2001)

Summary / Important Points for Beginners (I)



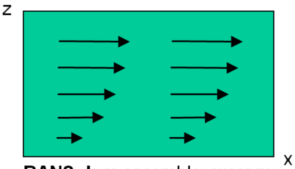
fluctuations
(**u**,c)



LES: volume average

_ _ _ critical concentration level

smooth result



RANS, k-ε: ensemble average

after Schatzmann and Leitl (2001)

Summary / Important Points for Beginners (II)

For an LES it always has to be guaranteed that the main energy containing eddies of the respective turbulent flow can really be simulated by the numerical model:

Summary / Important Points for Beginners (II)

For an LES it always has to be guaranteed that the main energy containing eddies of the respective turbulent flow can really be simulated by the numerical model:

- The grid spacing has to be fine enough.

Summary / Important Points for Beginners (II)

For an LES it always has to be guaranteed that the main energy containing eddies of the respective turbulent flow can really be simulated by the numerical model:

- The grid spacing has to be fine enough.
- $\mathrm{E}_{\mathrm{SGS}} < (<<)\ \mathrm{E}$

Summary / Important Points for Beginners (II)

For an LES it always has to be guaranteed that the main energy containing eddies of the respective turbulent flow can really be simulated by the numerical model:

- The grid spacing has to be fine enough.
- $\mathrm{E_{SGS}} < (<<)\ \mathrm{E}$
- The inflow/outflow boundaries must not effect the flow turbulence
  (therefore cyclic boundary conditions are used in most cases).

Summary / Important Points for Beginners (II)

For an LES it always has to be guaranteed that the main energy containing eddies of the respective turbulent flow can really be simulated by the numerical model:

- The grid spacing has to be fine enough.
- $\mathrm{E}_{\mathrm{SGS}} < (<<)\ \mathrm{E}$
- The inflow/outflow boundaries must not effect the flow turbulence
  (therefore cyclic boundary conditions are used in most cases).
- In case of homogeneous initial and boundary conditions, the onset of turbulence has to be triggered by imposing random fluctuations.

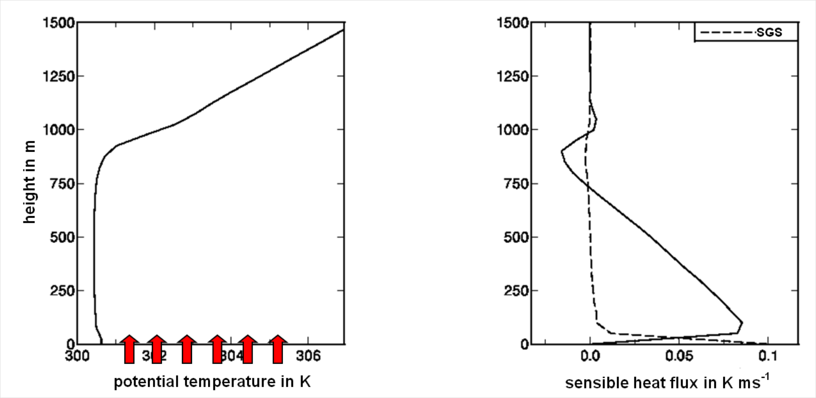
Summary / Important Points for Beginners (II)

For an LES it always has to be guaranteed that the main energy containing eddies of the respective turbulent flow can really be simulated by the numerical model:

- The grid spacing has to be fine enough.
- $\mathrm{E_{SGS}} < (<<)\ \mathrm{E}$
- The inflow/outflow boundaries must not effect the flow turbulence
  (therefore cyclic boundary conditions are used in most cases).
- In case of homogeneous initial and boundary conditions, the onset of turbulence has to be triggered by imposing random fluctuations.
- Simulations have to be run for a long time to reach a stationary state and stable statistics.

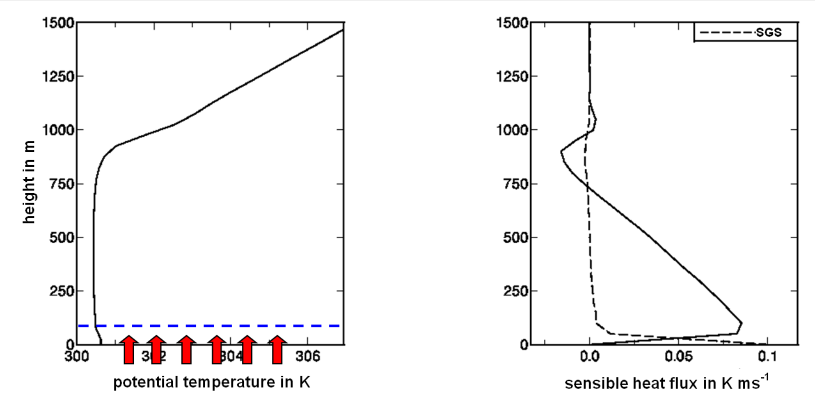
Example Output (I)

- LES of a convective boundary layer



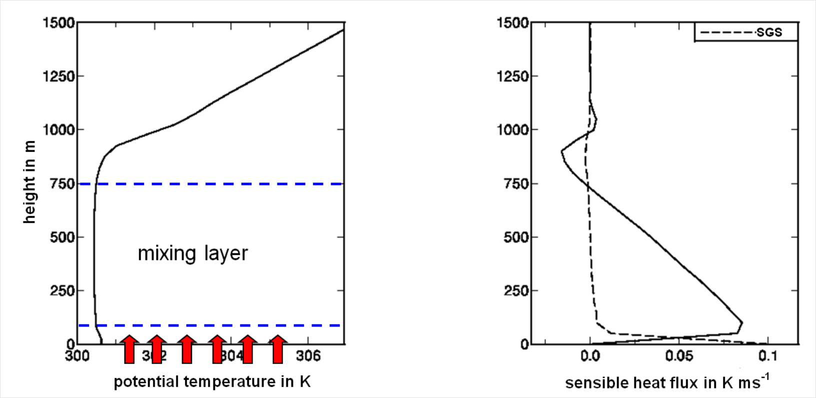
Example Output (I)

- LES of a convective boundary layer



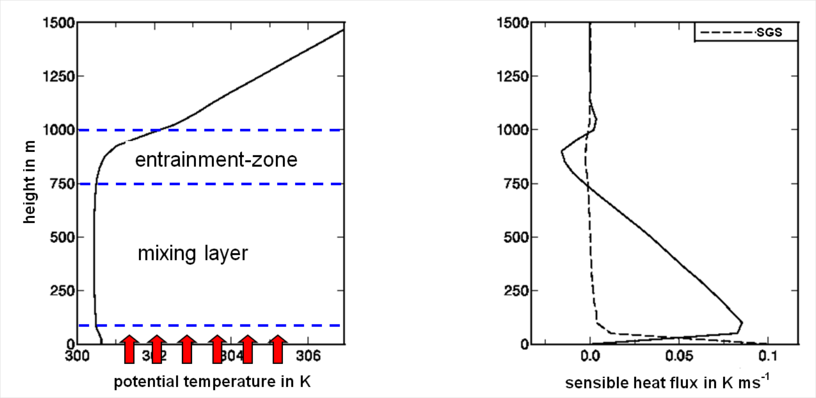
Example Output (I)

- LES of a convective boundary layer



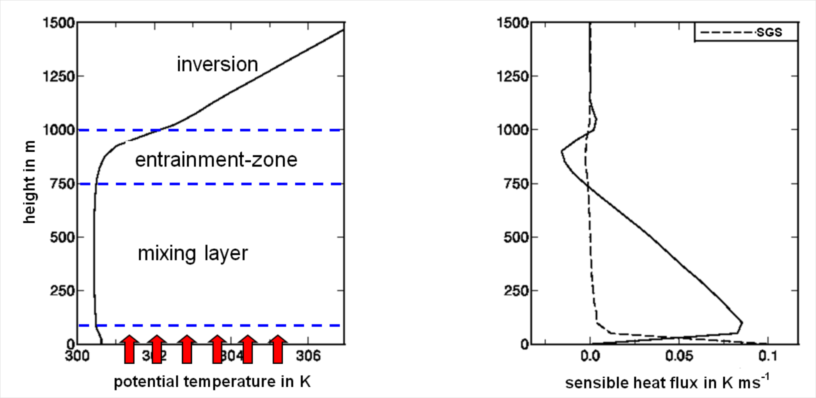
Example Output (I)

- LES of a convective boundary layer



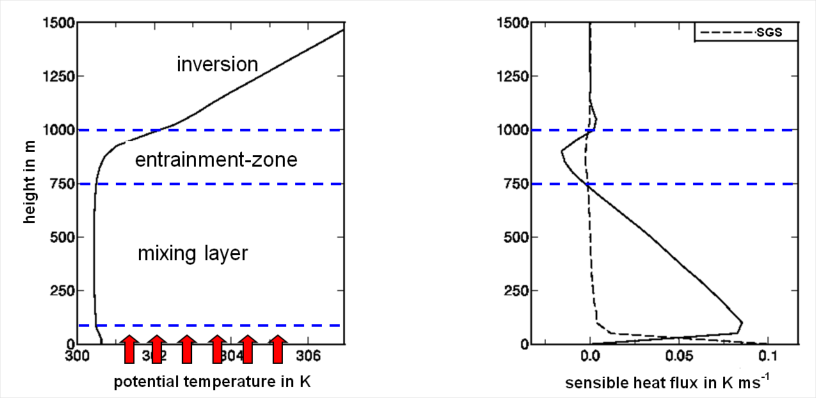
Example Output (I)

- LES of a convective boundary layer



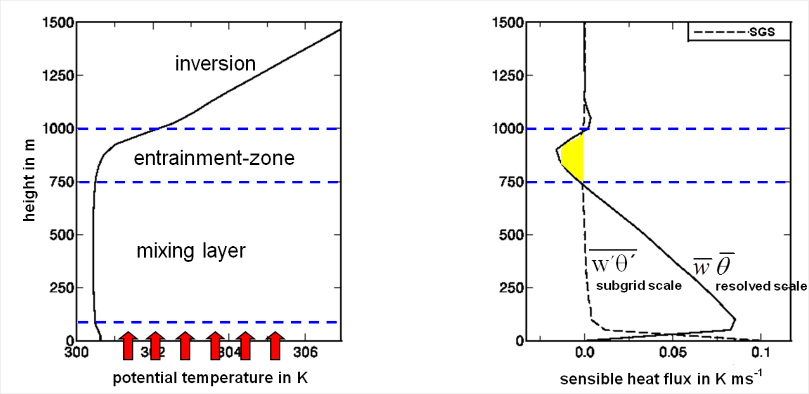
Example Output (I)

- LES of a convective boundary layer



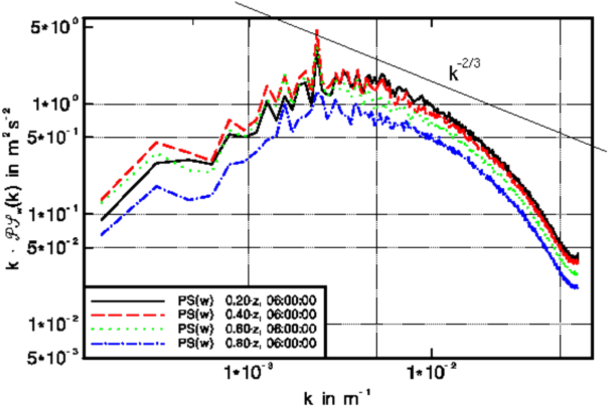
Example Output (I)

- LES of a convective boundary layer



Example Output (II)

- LES of a convective boundary layer



power spectrum of vertical velocity

Some Symbols

$u_i \quad (i = 1, 2, 3)$ u, v, w	velocity components
$x_i \quad (i = 1, 2, 3)$ x, y, z	spatial coordinates
Θ	potential temperature
Ψ	passive scalar
T	actual Temperatur
$\Phi = gz$	geopotential
p	pressure
ρ	density
f_i	Coriolis Parameter
ϵ_{ijk}	alternating symbol
ν, ν_Ψ	molecular diffusivity
Q, Q_Ψ	sources or sinks