Fundamentals of Large-Eddy Simulation

PALM group

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The Role of Turbulence (I)

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- **Characteristics of turbulence**
  - non-periodical, 3D stochastic movements
  - mixes air and its properties on scales between large-scale advection and molecular diffusion
  - non-linear → energy is distributed smoothly with wavelength
  - wide range of spatial and temporal scales

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  - Large scales: shear and buoyant production
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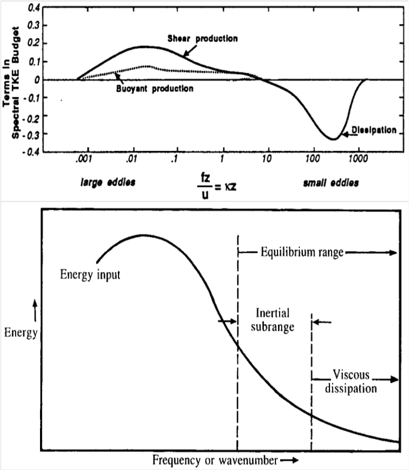

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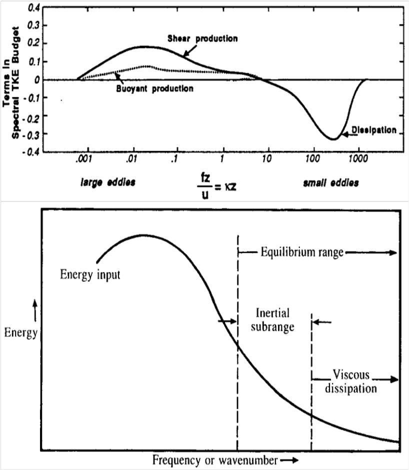

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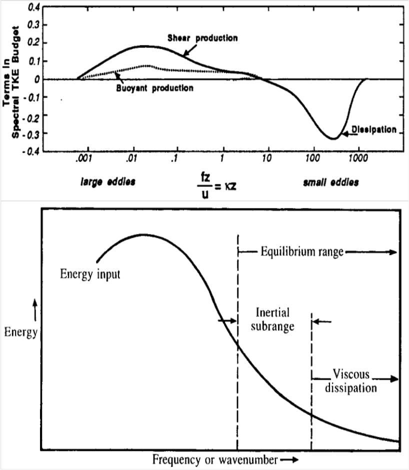
- **Energy-cascade**
  Large eddies are broken up by instabilities and their energy is handled down to smaller scales.



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The Reynolds Number (Re)

$\frac{L}{\eta} \approx Re^{3/4} \approx 10^6$ (in the atmosphere)

$\mathbf{u}$ 3D wind vector
ν kinematic molecular viscosity
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(Kolmogorov dissipation length)

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$\Rightarrow$ Number of gridpoints for a 3D simulation:

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  - **Consequences:**
    - DNS is restricted to moderately turbulent flows (low Reynolds-number flows).
    - Highly turbulent atmospheric turbulent flows cannot be simulated.

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  - **Consequence:**
    - Not suitable for detailed turbulence studies.

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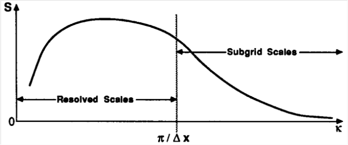
  - The impact of small eddies on the large-scale flow is parameterized.
  - Advantages:
    - Highly turbulent flows can be simulated.
    - Local homogeneity and isotropy at large *Re* (Kolmogorov's 1^{st} hypothesis) leaves parameterizations uniformly valid for a wide range of different flows.

Concept of Large Eddy Simulation (I)

- **Filtering**



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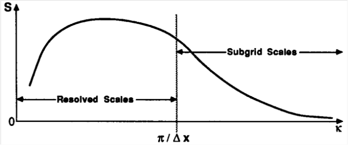

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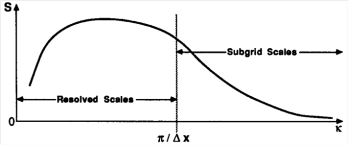

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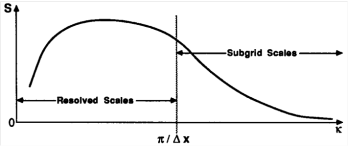

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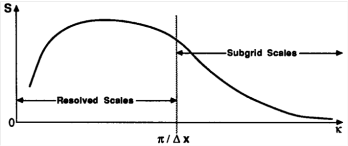

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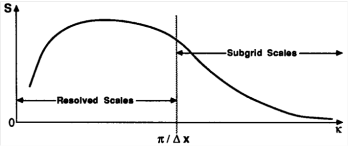
$$w = \overline{w} + w', \theta = \overline{\theta} + \theta'$$



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  - Therefore, these unknowns have to be parameterized using information from the resolved scales.
    - A typical example is the flux-gradient relationship, e.g.,

$$\overline{w'\theta'} = -\nu_\mathrm{h} \cdot \frac{\partial\overline{\theta}}{\partial z}$$